Quiz 19

November 11, 2016

Show all work and circle your final answer.

1. *(5 points)* Find the points on $y = x^2 + 1$ closest to the point $(0, 2)$.

$d^2 = (x-0)^2 + (y-2)^2$

$D = d^2 = x^2 + (x^2+1-2)^2$

$D = d^2 = x^2 + (x^2-1)^2$

$\frac{dD}{dx} = 2x + 2(x^2-1)(2x) \overset{\text{set}}{=} 0$

$2x(1 + 2x^2 - 2) = 0$

$2x(2x^2-1) = 0$

$x = 0, \pm\sqrt{1/2}$

$D'' = 12x^2 - 2$

$D''(0) < 0$ so max

$D''(\pm\sqrt{1/2}) > 0$ so mins

The distance from $(\sqrt{1/2}, 3/2)$ to $(0,2)$ is the same as the distance from $(-\sqrt{1/2}, 3/2)$ to $(0,2)$.

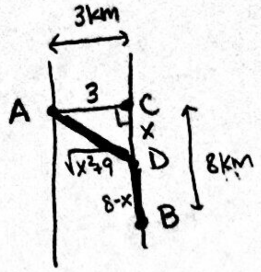
$\boxed{(\pm\sqrt{1/2}, 3/2)}$

2. *(15 points)* Ali launches her canoe from point A on a bank of a 3 km wide river, and wants to reach point B, 8 km downstream on the opposite bank, as quickly as possible. She can proceed in any of three ways:

   1. Ròw her boat directly across the river to point C and then run to B
   2. Row directly to B
   3. Row to some point D between C and B and then run to B

   If she can row 6 km/h and run 8 km/h, where should she land to reach B as soon as possible?

   Hint: distance = rate × time can be rewritten as time $= \dfrac{\text{distance}}{\text{rate}}$.



Minimize time:

$$t = \frac{\sqrt{x^2+9}}{6} + \frac{8-x}{8}$$

$$t' = \frac{1}{6}\left[\frac{1}{2}(x^2+9)^{-1/2}(2x)\right] - \frac{1}{8}$$

$$= \frac{x}{6\sqrt{x^2+9}} - \frac{1}{8} \overset{set}{=} 0$$

$0 \le x \le 8$

$$\frac{x}{6\sqrt{x^2+9}} = \frac{1}{8}$$

$$x = \frac{3}{4}\sqrt{x^2+9}$$

Square both sides: $x^2 = \frac{9}{16}(x^2+9)$

$$x^2 = \frac{9}{16}x^2 + \frac{81}{16}$$

$$\frac{7}{16}x^2 = \frac{81}{16}$$

$$x^2 = \frac{81}{7}$$

$$x = \pm 9/\sqrt{7}$$

Check: $t''(x) = \dfrac{6\sqrt{x^2+9} - \frac{6x}{2}\left(\frac{1}{\sqrt{x^2+9}}\right)(2x)}{36(x^2+9)} \cdot \dfrac{\sqrt{x^2+9}}{\sqrt{x^2+9}}$

$$= \frac{6(x^2+9) - 6x^2}{36(x^2+9)^{3/2}}$$

$$= \frac{54}{36(x^2+9)^{3/2}}$$

$t''(9/\sqrt{7}) > 0$, so $\boxed{x = 9/\sqrt{7}}$ minimizes t.